Problem 1. Pure Nash equilibria

Two tobacco companies must decide whether to buy ads for the next Formula 1 race.

- Displaying ads gives you a marketing advantage over the other producer.
- Total sales are marginally affected by advertising (the number of people smoking is the same, they only move from one brand to another).
- The total value of sales is 4. Advertising costs 0.5 (see matrix above).

	No Ad	Ad
No Ad	$(2, 2)$	$(1, 2.5)$
Ad	$(2.5, 1)$	$(1.5, 1.5)$

a) What is the Nash equilibrium of this game?

b) Formula 1 organizers now forbid tobacco advertising in their races. Are tobacco companies going to complain?

Solution:

a) The game's Nash Equilibrium is (Ad, Ad). If we suppose company 1 buys ads, then company 2 receives more revenue by buying ads than by not buying ads. Similarly, if we suppose company 2 buys ads, then company 1 receives more revenue by buying ads than by not buying ads. Hence, (Ad, Ad) is a Nash Equilibrium.

b) If the organizers forbid tobacco advertising, then both companies must play (No Ad, No Ad). Since $(2, 2) \succ (1.5, 1.5)$, the payoff from (No Ad, No Ad) is greater for both companies than the payoff from the Nash Equilibrium (Ad, Ad), and so neither will complain.

Additional problems: Solve Exercises 1 and 2 in the slides 01-Static games.pdf.

Solution of Exercise 1 (Iterated elimination of dominated actions)

- No action is dominated or weakly dominated for player 1. For player 2, action y is strictly dominated by z and w. Moreover, w is weakly dominated by z. Hence, we can remove actions y and w.
- The reduced payoff matrix is given by:

	x	z
a	$(-1, 1)$	$(3, -3)$
b	$(1, -1)$	$(2, -2)$
c	$(4, -4)$	$(3, -3)$

  In the reduced payoff matrix, action c of player 1 is dominated by b (and weakly dominated by a). Thus, we can remove action c and we then find that action x of player 2 is dominated by z. Lastly, we have that action a is dominated by action b. This leaves action profile (b, z) which is a Nash equilibrium. Notice that by removing weakly dominated actions we removed the following Nash equilibrium: (b, w).

- In the case where $A = B$, action z and w are dominated by y. We can not remove any other action. So, action dominance cannot lead to a unique outcome of the game. However, From the reduced game, it is easier to find the Nash equilibria: (a, x) and (c, y).

We observe that action dominance cannot always lead to a unique outcome of the game. However, it can help remove some actions and form a reduced game, which would be in turn simpler to analyze.

Solution of Exercise 2 (The Stag Hunt)

a) The payoff matrix is given by:

$$\begin{array}{c c} & \begin{array}{cc} \text{Stag} & \text{Hare} \end{array} \\ \begin{array}{c} \text{Stag} \\ \text{Hare} \end{array} & \left[\begin{array}{cc} (\frac{D}{2}, \frac{D}{2}) & (0, R) \\ (R, 0) & (\frac{R}{2}, \frac{R}{2}) \end{array} \right]. \end{array}$$

b) There is no dominated action. In fact $\frac{D}{2} > R$, but $0 < \frac{R}{2}$.

c) The security level is given by

$$\max_{a \in \{Stag, Hare\}} \min_{b \in \{Stag, Hare\}} J_{a,b} = \max\left(\min\left(\frac{D}{2}, 0\right), \min\left(R, \frac{R}{2}\right)\right) = \max\left(0, \frac{R}{2}\right)$$

The security strategy to obtain the security level is *Hare*.

d) There are two Nash equilibria: (Stag, Stag) and (Hare, Hare).